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Conductance oscillations related to the density of states of a circular quantum dot in weak magnetic fields

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Abstract. The conductance of a quantum dot has been measured as a function of the magnetic field and the size of the dot. The dot was formed by electrostatic depletion in a GaAs/GaAlAs heterostructure and contained approximately 1700 electrons. Measured conductance variations agree with calculated values of a simplified model, where electrons tunnel through the energy levels of an isolated perfectly circular dot. The maximum peak amplitudes in the Fourier power spectra of the magnetoconductance decrease stepwise, with steps at multiples of the Aharonov–Bohm frequency $f = eA/h$, where A is the dot area.

The transport behaviour of ballistic microstructures can be theoretically analysed using billiard-ball models, where electrons move as classical particles and bounce at the hard confining walls [1, 2]. The geometric form and small irregularities of the confining potential have then been found to make large changes in the path for such a classical ballistic particle. For the case of non-integrable geometries, the classical scattering dynamics are therefore chaotic in the sense that an orbit in the dot is extremely sensitive to its initial conditions. The statistical properties of such fluctuations have been predicted from scattering dynamics [1] and investigated experimentally by Marcus *et al* [3]. Stone and Bruus [2] have found that minor distortions of the boundary may dramatically change the energy levels of a quantum dot, indicating that an idealized model geometry, such as a circle, is not relevant for describing an experimental system.

The quantum behaviour is introduced by the Bohr–Sommerfeld quantization rule. When electrons move in closed trajectories in a magnetic field, the phase change along the loop is determined by the enclosed flux and the conductance will be a periodic function of magnetic flux with period $\Phi_0 = h/e$.

We report regular variations of the measured conductance of a dot with a roughly circular shape. The conductance is theoretically found to be proportional to the density of states at the Fermi energy, which for a circular dot is related to the zeros of Bessel functions. Using such an idealized model we find similar variations in both measurements and calculations of the conductance. Conductance maxima recurrently appear as a function of gate voltage at zero magnetic field. They are related to the zeros of the first- and zeroth-order Bessel functions, but can also be interpreted semiclassically, from the Bohr–Sommerfeld quantization of a two-bounce orbit for an electron moving back and forth in the dot. States related to higher-order Bessel functions have in general a stronger magnetic field dependence. We show

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experimentally that Fourier power spectra of the magnetoconductance have several peaks with high amplitudes below the frequency $1/\Delta B = \pi a^2 e/h$. The period ΔB corresponds to the addition of one magnetic flux quantum inside the dot periphery. At higher frequencies the peaks are one order of magnitude smaller. The experiments agree with the simple model of a perfectly circular disc, despite the fact that impurities and contacts that make the confinement irregular. The conductance variations are due to clustering of the energy levels, to a kind of shell structure.

The experiment was performed on a quantum dot, confined in a two-dimensional electron gas (2DEG) by the electrostatic potential of four Schottky gates and connected to the environment by two quantum point contacts. The 2DEG was formed in a GaAs/$Ga_xAl_{1-x}As$ heterostructure. The mobility, 63 m^2 V^{-1} s^{-1}, and sheet electron density, $n_s = 3.7 \times 10^{15}$ m^{-2}, were determined by low-magnetic-field Hall and resistivity measurements at $T = 9$ K. The elastic mean free path is estimated to be 6 μm. The gate geometry, shown in the left inset of figure 1, had a lithographically defined inner diameter of 1 μm. The diameter, $2a$, of the quantum dot may be estimated to be 0.8 μm, if the depletion width is taken into account. The number of electrons in the dot was roughly 1700.

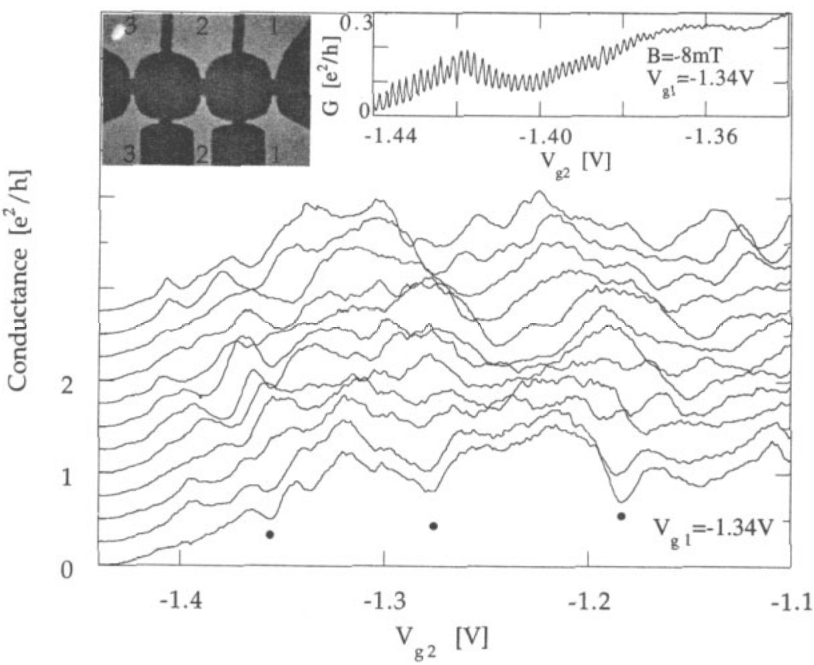

Figure 1. Conductance oscillations as a function of gate voltage. The magnetic field is changed from 0 mT (bottom trace) to 48 mT (top trace), in steps of 8 mT (traces are offset for clarity). The Coulomb blockade oscillations are smeared by a long integration time on the lock-in amplifiers. The left inset shows schematically the gate geometry with the two gate pairs. The dots indicate recurrently appearing conductance minima at $B = 0$ mT. Right inset: Coulomb blockade oscillations appear with a slower sweep rate if the conductances of both point contacts are lower than $2e^2/h$. $B = 8$ mT. The gate voltage settings are the same as in the left inset. $T \approx 20$ mK.

We determined the two-terminal conductance by measuring the current and the voltage across the sample with two lock-in amplifiers, while the magnetic field or the gate voltage on one gate pair (V_{g1}) was swept. The dot was biased symmetrically by an AC voltage over two 100 kΩ resistors, connected in series with the sample. The AC voltage was less than 10 μV. The measurements were made in an He^3/He^4 dilution cryostat at its base temperature, $T \approx 20$ mK. Due to insufficient thermal contact to the sample the effective sample temperature is estimated to have been in the range 100–200 mK. This estimate is based on observations of the amplitudes of the Coulomb blockade oscillation peaks [4]. The enhanced sample temperature is most likely due to room temperature radiation reaching the sample.

The conductance of the dot is shown in figure 1 as a function of V_{g1} at different magnetic fields. The number of one-dimensional modes in the point contact changes from zero to about two in these gate voltage sweeps. The total conductance changes from pinch-off to around e^2/h. The trace has recurrently appearing minima and maxima. The three approximately equidistant minima are marked with dots in the figure. Conductance features move systematically with increasing magnetic field. Coulomb blockade oscillations [5] were observed, when both point contacts had a conductance lower than $2e^2/h$. The right inset of figure 1 shows a trace with the same gate voltage settings, but taken with a slower sweep rate, allowing blockade oscillations to be seen. This measurement gave an average blockade period of 1.7 mV, increasing, however, for increasing gate voltage.

To illustrate the overall regular variation more clearly, we display a grey scale image of the measured conductance using a large number of gate voltage sweeps in figure 2(a), with magnetic field intervals of 1 mT. A third-order polynomial is subtracted from each sweep to make the oscillatory behaviour more visible and remove the effect of the varying conductance of the point contact. Bright regions correspond to high conductance. The three minima, marked with dots in figure 1(a), can be recognized as dark spots at zero magnetic field in the grey scale figure 2(a). Weak diagonal wavelike patterns can be distinguished at the top of figure 2(a).

To find the origin of the measured conductance variations, we calculated the conductance of a simple model geometry when the dot size and the magnetic field are varied. The result is displayed as a grey scale image in figure 3(a). The calculation is based on a transport model where the electrons enter from one lead and tunnel through levels near the Fermi energy to the other lead. The calculated conductance variations reflect changes in the density of states at the Fermi energy. Features of the measured data in figure 2(a) are retrieved in figure 3(a). We now turn to a description of the calculation before further comparisons are made.

The model geometry is a perfectly circular disc with hard walls. The Schrödinger equation is solved for weak magnetic fields, by perturbation theory [6, 7]. If the applied magnetic field is sufficiently small ($B \ll 0.25$ T), such that the cyclotron radius is much larger than the dot radius a, the energy levels can be expressed as

$$E_{n,m} = E_0[\gamma_{n,m}^2 + 2n\alpha + \tfrac{1}{3}\alpha^2(1 + 2(n^2 - 1)/\gamma_{n,m}^2)] \tag{1}$$

where $E_0 = \hbar^2/2m^*a^2$, $\gamma_{n,m}$ is the mth root of the nth-order Bessel function, $J_n(\gamma_{n,m}) = 0$, $n = 0, \pm1, \pm2, \ldots$, $m = 1, 2, \ldots$. For a given magnetic field, the flux through the dot is $\Phi = \pi a^2 B$, which can be normalized by the magnetic flux quantum $\Phi_0 = h/e$, giving the dimensionless parameter $\alpha = \Phi/\Phi_0$.

We cannot expect to resolve single energy levels, but clustering of many levels near the Fermi energy will result in larger conductance. To calculate the conductance we use a Landauer-type formula [8]

$$G = \frac{2e^2}{h}\int \mathrm{d}E \left(-\frac{\mathrm{d}f}{\mathrm{d}E}\right) T(E) \propto \frac{e^2\Gamma}{\hbar k_B T}\sum_{n,m}\frac{\eta_{n,m}}{(1+\eta_{n,m})^2} \tag{2}$$

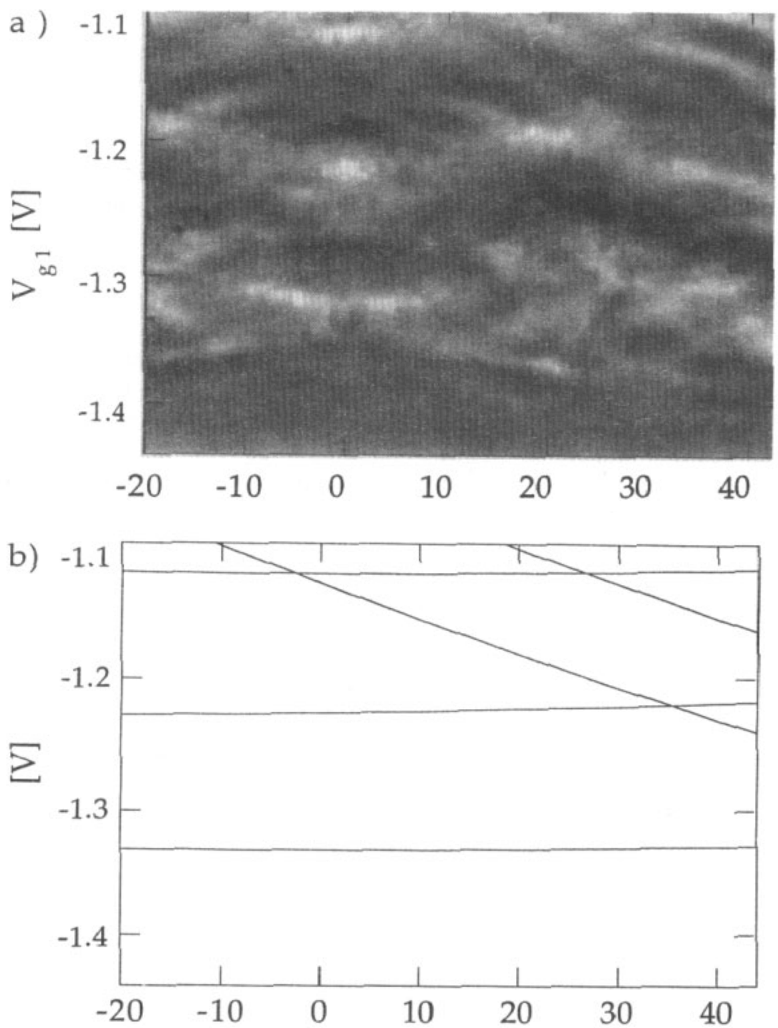

Figure 2. (a) A grey scale image of the measured conductance as a function of gate voltage and magnetic field. Bright areas indicate high conductance. B was changed in steps of 1 mT. $V_{g2} = -1.34$ V. The minima marked with dots in figure 1(a) can be seen at $B = 0$ mT as dark regions. (b) The lines indicate resonances to single levels (cf figure 3(b)), where high density of states is expected. The distance between them is fitted to the high-conductance regions in (a) relating a change of the gate voltage to a change of radius. The horizontal lines indicate levels with $n, m = 0, 18; 1, 17; 0, 17$ (from the top) and the diagonal lines, $n, m = -24, 8; -23, 8$.

where $\eta_{n,m} = \exp[(E_{n,m} - E_F)/k_BT]$ and $T(E)$ is the transmission probability for states at the energy E. We assume that the transmission probability is equal for all states and the thermal broadening k_BT is larger than the Lorentzian broadening Γ of the levels. Equation (2) relates the conductance to the density of states of the dot, with the thermal smearing determined by the derivative of the Fermi–Dirac function $\mathrm{d}f/\mathrm{d}E$.

We may assume that the major effect of changing the gate voltage is a change of size of the dot, for a lithographically defined diameter much larger than the gate depletion width. The conductance is therefore calculated, with a constant Fermi energy $E_F = 13.2$ meV,

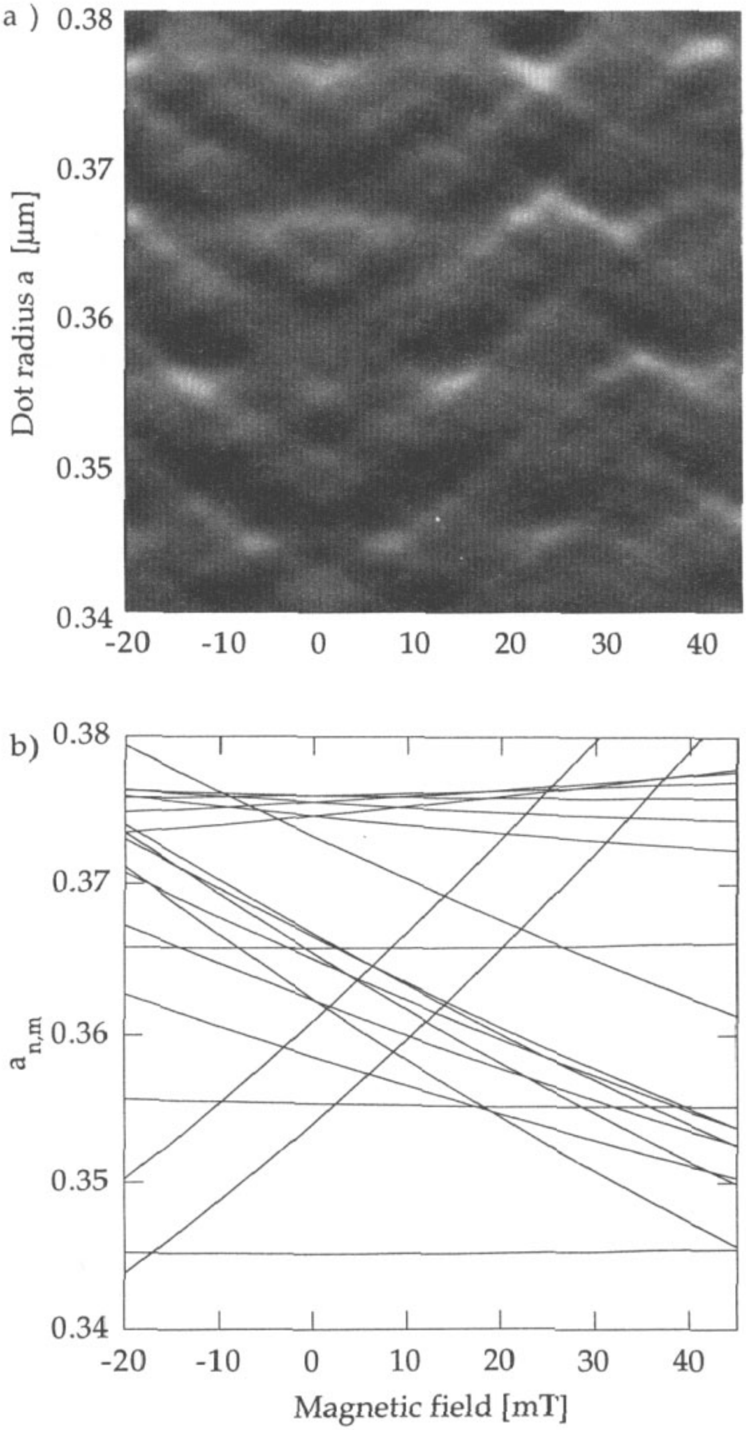

Figure 3. (a) The calculated conductance as a function of radius and magnetic field. (b) The lines indicate a few of the resonances for which $E_{nm} = E_F$, for a dot with $E_F = 13.2$ meV. The levels with low quantum number n are clustered along lines, caustics, with weak magnetic field dependence. At the top a_{nm}: $n, m = 1, 18; -1, 18; 3, 17; -3, 17; 5, 16; -5, 16$ are clustered to such a caustic and below them a_{nm}: $0, 18; -1, 17; 0, 17$ indicate the position of other regions with high density of states. The levels with a high n number (a_{nm}: $-17, 10; -20, 9; -23, 8; -26, 7; -29, 6; -32, 5; -35, 4$ and $-24, 8$) have a strong magnetic field dependence and are clustered along diagonal caustics. The levels with highest n ($m = 1$) have the strongest magnetic field dependence; a_{nm}: 48, 1 and 47, 1 are drawn.

and displayed as a function of dot radius and magnetic field in the grey scale image in figure 3(a). The radius $a_{n,m}$, for which the resonance condition $E_{n,m} = E_F$ is satisfied, can be deduced from (1). Lines corresponding to a few single-level resonances are drawn in figure 3(b). *Notice that a resonance of a single level is not seen in the grey scale images*—bright regions are only seen where a large number of levels have the same or nearly the same energy.

The low-order single-level resonances have nearly the same energy near $B = 0$ T, where the density of states consequently is high. These energies with high density of states appear recurrently as the quantum number m increases and result in a regularity of conductance variations at zero magnetic field. Low-order resonance lines ($n = 0$ and 1) from figure 3(b) are fitted to the gate voltage of the experimental data in figure 2(b), where they indicate three of the conductance maxima at zero magnetic field. The fit gives the change of radius Δr induced by ΔV_{g1} to be $\Delta r/\Delta V_{g1} = 0.10$ μm V^{-1} ($a = 0.4$ μm). This is comparable to the relation $\Delta r/\Delta V_{g1} = 0.06$ μm V^{-1} obtained by assuming the area to increase by $1/n_s$ as one electron is added to the dot, i.e., each 1.7 mV Coulomb blockade period.

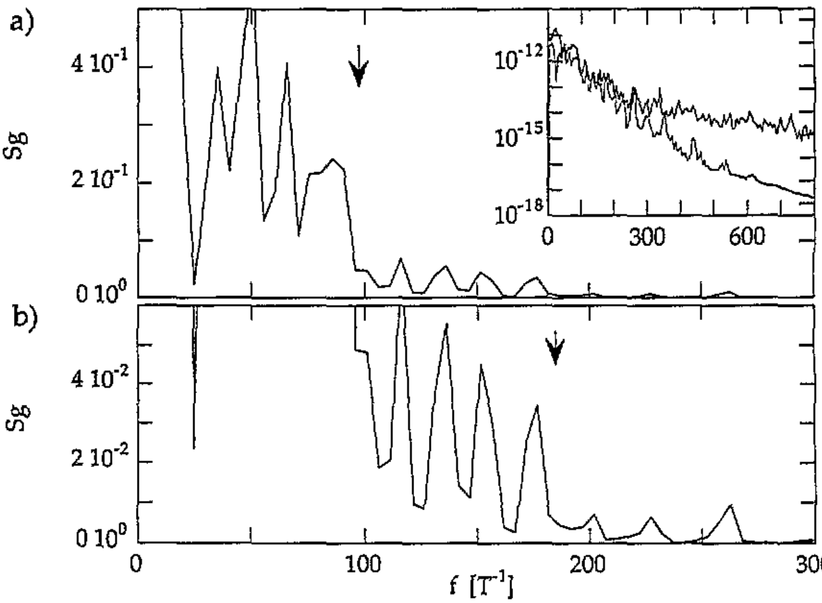

Figure 4. (a) The maximum amplitudes of 25 numerical Fourier power spectra of the magnetoconductance ($B = -100$–100 mT), where the radius is changed from 0.36 to 0.37 μm (arbitrary y scale). $T = 400$ mK. (b) The same as (a) with an expanded y scale. A first drop of amplitude appears at 100 T^{-1}, indicated with an arrow. A few peaks are seen between 100 and 200 T^{-1}, then another drop appears (indicated with an arrow in (b)). The inset of (a) shows both the experimental and calculated (normalized) power spectra on a logarithmic y scale.

High-order levels in general vary more rapidly with magnetic field. They cluster at different ranges in parameter space and cause a diagonal wave pattern in the grey scale images. This is exemplified in figure 3(b), where lines are drawn for the first root of two highest-order levels ($n, m = 47, 1; 48, 1$) and a number of intermediate levels with a smaller slope. These intermediate levels are also drawn in figure 2(b), where they coincide with the observed diagonal pattern.

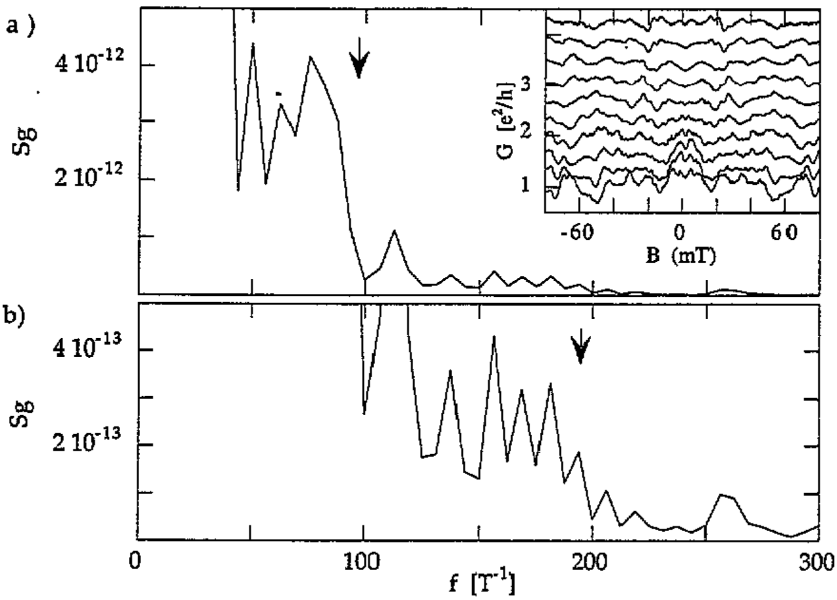

Figure 5. (a) The maximum amplitude of 20 experimental Fourier power spectra of the magnetic field sweeps ($B = -80$–80 mT), where the gate voltage is changed in steps of 2 mV, from -1.410 V to -1.372 V. $V_{g2} = -1.310$ V. $T \approx 20$ mK. The amplitude decreases by one order of magnitude at a frequency 100 T^{-1}, indicated with an arrow. (b) The linear y scale is expanded and a second drop is seen at 200 T^{-1}, indicated with an arrow. The inset shows every other one of these sweeps, which are offset for clarity. $V_{g1} = -1.410$ V in the top trace.

The conductance will vary non-monotonically when the magnetic field changes. The shortest period in the level spectrum is between consecutive first roots of high order n. The distances between other roots that simultaneously contribute to conductance variations cause oscillations with longer periods. This is reflected in the Fourier transform power spectra of magnetoconductance shown in figure 4, where the amplitude is large for frequencies lower than a cut-off frequency $f = \pi a^2 e/h$, corresponding to additions of one magnetic flux quantum to the dot (for $a = 0.36$ μm $f \approx 100$ T^{-1}). The origin of this cut-off frequency can also be understood in a semiclassical picture. States bouncing along the perimeter of the dot enclose an area approaching the full dot area and result in periodic oscillations with this frequency. States with fewer bounces (down to two) enclose smaller areas and result in peaks at frequencies lower than f.

Power spectra of experimental magnetic field sweeps, $B = -0.08$–0.08 T, have a number of peaks with highest amplitudes at low frequencies, below 100 T^{-1}. The amplitudes of some peaks increase while others decrease when the gate voltage is slightly changed. Figure 4 shows numerical power spectra and figure 5 experimental spectra, where we have plotted the maximum values of several spectra within a small range of radius and gate voltage. The amplitude drops with about one order of magnitude at about 100 T^{-1}. The amplitudes again drop one order of magnitude at $f \approx 200$ T^{-1}. These steps are observed in both experimental and numerical spectra.

Fourier power spectra of magnetoconductance oscillations have been analysed theoretically, for both chaotic and integrable systems, by Jalabert *et al* [1]. They semiclassically predict an exponential decay of amplitudes of the power spectra for a chaotic system [1],

$$S_g(f) = S_g(0)[1 + (2\pi\beta\Phi_0)f]e^{-(2\pi\Phi_0)f} \tag{3}$$

where f is the frequency and β is a constant. A spectrum of a chaotic geometry in the form of a stadium was calculated and correspondence with (3) was found over three orders of magnitude of the amplitude.

The inset of figure 4(a) shows both the experimental and the numerical power spectra for circular (non-chaotic) geometry on a logarithmic scale, and they both show a generally exponential decay. The experimental spectrum flattens out at frequencies higher than 250 T^{-1}, but the numerical spectrum continues to decrease exponentially over five orders of magnitude. The semiclassical prediction (3) for a chaotic system yields the same result as this calculation of a perfectly circular geometry, except for the observed steps at low frequencies seen in figures 4 and 5.

In conclusion, conductance variations of a quantum dot in weak magnetic fields are experimentally found to originate from changes in its density of states at the Fermi energy, even for a rather open structure with conductance of the order of e^2/h. The energy spectrum is found to depend on the magnetic field and the size of the dot in a regular manner. The measured transport properties agreed with a tunnelling model where energy levels were calculated for a circular dot. The recurrently appearing high conductance at zero field, and the diagonal pattern in conductance versus gate voltage and magnetic field, can be explained as a coincidence of many single levels. The high conductance of the point contacts makes it possible to compare level statistics related to the geometry with transport measurements, without taking charging effects into account.

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